Checking Joe Boninger's conjecture that for a closure of a positive braid the coefficients of ρ_1 are all positive.

pdf

In[ ]:=

```
SetDirectory["C:\\Users\\T15Roland\\Wiskunde\\Bn\\Heisenberg"];
Once[<< KnotTheory`; << Rot.m];
```

pdf

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

pdf

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

pdf

ToFileName: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}].

pdf

ToFileName: String or list of strings expected at position 1 in ToFileName[{File, QuantumGroups}].

pdf

```
Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

```
Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

pdf

In[ ]:=

```
R1[s_, i_, j_] := s (g[j,i] (g[j+1,j] + g[j,j+1] - g[i,j]) - g[i,i] (g[j,j+1] - 1) - 1/2);
ρ[K_] := ρ[K] = Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ( {{-T^s, T^s - 1}, {0, -1}} ))];
   Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
   G = Inverse[A];
   ρ1 = Sum[R1 @@ Cs[[k]], {k, 1, n}] - Sum[φ[[k]] (g[k,k] - 1/2), {k, 1, 2 n}];
   Factor@{Δ, Δ^2 ρ1 /. g[α_, β_] :> G[[α, β]]}];
```

```
In[•]:= Mat[u_] :=
  Transpose@Table[Prepend[Table[Coefficient[(x^(1/2) - x^(-1/2))^(2 j) // Expand, x^k], {k, 1, u}],
     (Expand[(x^(1/2) - x^(-1/2))^(2 j)] /. {x^(a_Integer) :> 0, x -> 0})], {j, 0, u}]
ToConway[P_] :=
  If[P === 0, 0, Module[{CC, deg = Exponent[P, T], M}, M = Inverse[Mat[deg]];
    CC = Expand[P] /. {T^(j_) :>
        If[j < 0, 0, T^j]};
    Total[CoefficientRules[CC, T]] /.
      {({a_} -> b_) :> b M[[;; , a + 1]].Table[z^(2 i), {i, 0, deg}]} // Expand
   ]]

ToConway[7 + 3 / T^2 + 3 T^2 - 101 T^4 - 101 / T^4]
% /. {z -> x^(1/2) - x^(-1/2)} // Expand
```

Out[•]=

$-189 - 1604 z^2 - 2017 z^4 - 808 z^6 - 101 z^8$

Out[•]=

$7 - \frac{101}{x^4} + \frac{3}{x^2} + 3 x^2 - 101 x^4$

```
In[•]:= Cρ1[K_] := ToConway[Expand@Together[ρ[K][[2]] (-T) / (-1 + T)^2]]
Cρ0[K_] := ToConway[Expand@Together[ρ[K][[1]]]]
CheckForConstantSigns[P_] :=
  If[Length@Union[Sign /@ (MonomialList[P, z] /. z -> 1)] == 1, True, False]
PosKnots_i_ [n_] :=
  Table[If[CheckForConstantSigns[Cρ_i[K]], K, Nothing], {K, AllKnots[{3, n}]}]
```

```
In[•]:= PosKnots1[10];
```

```
In[•]:= PosKnots0[10];
```

```
In[•]:= (*False Alexander positives detected by ρ1,
i.e. knots that are not positive braid closures where the Alex is positive.*)
Complement[PosKnots0[10], PosKnots1[10]]
```

Out[•]=

{Knot[9, 26], Knot[9, 29], Knot[9, 41], Knot[9, 44], Knot[10, 93],
 Knot[10, 108], Knot[10, 113], Knot[10, 129], Knot[10, 146], Knot[10, 164]}

```
In[•]:= {Cρ1[#], Cρ0[#]} &@Knot[9, 41]
```

Out[•]=

$\{2 - 17 z^2 + 2 z^4 - 3 z^6, 1 + 3 z^4\}$

In[•]:=
```
(*Copied from KnotInfo*)
PosBraidKnots = {Knot[3, 1], Knot[5, 1], Knot[7, 1], Knot[8, 19], Knot[9, 1],
   Knot[10, 124], Knot[10, 139], Knot[10, 152], Knot[11, Alternating, 367],
   Knot[11, NonAlternating, 77], Knot[12, NonAlternating, 242],
   Knot[12, NonAlternating, 472], Knot[12, NonAlternating, 574],
   Knot[12, NonAlternating, 679], Knot[12, NonAlternating, 688],
   Knot[12, NonAlternating, 725], Knot[12, NonAlternating, 888]};

(*Checking Boninger's conjecture up to 12 crossings*)
CheckForConstantSigns[Cρ1[#]] & /@ PosBraidKnots
```

Out[•]=
```
{True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True}
```

In[◦]:=
```
(*Values of normalized rho2 copied from the table in PG.pdf*)
ρ2819 = ToConway[3 T^(11) + T^(10) - 2 T^9 + 17 T ^8 + 14 T ^7 -
    29 T ^6 + 77 T^ 5 - 10 T^ 4 + 65 T ^3 + 29 T ^2 - 72 T + 239 /. {T → T + T^-1}]
ρ291 = ToConway[4 T ^(15) + T^( 14) + 13 T^( 13) + 34 T^( 11) - 14 T^( 10) + 84 T ^9 - 64 T^ 8 +
    194 T^ 7 - 54 T^ 6 + 161 T^ 5 + 137 T^ 4 - 52 T ^3 + 462 T^ 2 - 312 T + 702 /. {T → T + T^-1}]
ρ210124 =
 ToConway[4 T ^(15) + T^( 14) - 3 T^( 13) + 21 T^( 12) - 12 T ^(11) + 42 T^( 10) + 13 T^( 9) -
    61 T^8 + 169 T^ 7 + 11 T^ 6 - 21 T^ 5 + 45 T ^4 + 278 T^ 3 - 89 T^ 2 - 168 T + 604 /. {T → T + T^-1}]
ρ210139 =
 ToConway[4 T ^(15) + T^( 14) - 3 T^( 13) + 42 T^( 12) - 54 T^( 11) + 73 T^( 10) + 20 T ^9 -
    9 T ^8 - 16 T^ 7 + 328 T^ 6 - 209 T^ 5 - 22 T^ 4 + 619 T^ 3 - 306 T^ 2 - 72 T + 677
   /. {T → T + T^-1}]
ρ210139 =
 ToConway[4 T ^(15) + T^( 14) - 19 T^( 13) + 63 T^( 12) - 66 T^( 11) + 54 T^( 10) - 21 T^ 9 +
    248 T^ 8 - 755 T^ 7 + 1352 T^ 6 - 960 T^ 5 - 420 T^ 4 + 1696 T^ 3 - 764 T^ 2 - 1612 T + 3089
   /. {T → T + T^-1}]
```

Out[◦]=

$$13467 + 59080 z^2 + 126035 z^4 + 168057 z^6 + 154076 z^8 + 101257 z^{10} + 48439 z^{12} + 16798 z^{14} + 4121 z^{16} + 678 z^{18} + 67 z^{20} + 3 z^{22}$$

Out[◦]=

$$366102 + 2323968 z^2 + 7092318 z^4 + 13672924 z^6 + 18533979 z^8 + 18657345 z^{10} + 14381990 z^{12} + 8633794 z^{14} + 4066112 z^{16} + 1501348 z^{18} + 430878 z^{20} + 94362 z^{22} + 15262 z^{24} + 1721 z^{26} + 121 z^{28} + 4 z^{30}$$

Out[◦]=

$$242888 + 1581836 z^2 + 4967107 z^4 + 9882054 z^6 + 13859495 z^8 + 14462851 z^{10} + 11569913 z^{12} + 7209513 z^{14} + 3521573 z^{16} + 1346197 z^{18} + 398858 z^{20} + 89828 z^{22} + 14867 z^{24} + 1705 z^{26} + 121 z^{28} + 4 z^{30}$$

Out[◦]=

$$283173 + 1817060 z^2 + 5623152 z^4 + 11019795 z^6 + 15214912 z^8 + 15625359 z^{10} + 12303256 z^{12} + 7551392 z^{14} + 3638211 z^{16} + 1374544 z^{18} + 403509 z^{20} + 90290 z^{22} + 14888 z^{24} + 1705 z^{26} + 121 z^{28} + 4 z^{30}$$

Out[◦]=

$$185545 + 1219716 z^2 + 3886852 z^4 + 7858672 z^6 + 11225652 z^8 + 11956612 z^{10} + 9781406 z^{12} + 6242301 z^{14} + 3125846 z^{16} + 1225403 z^{18} + 372162 z^{20} + 85790 z^{22} + 14493 z^{24} + 1689 z^{26} + 121 z^{28} + 4 z^{30}$$